Affine Springer fibers, Cherednik algebras and Hilbert schemes

The aim of this reading group is to introduce the participants to some of the basic objects of the affine Springer theory and connections of this theory to Hilbert schemes of points on surfaces, Cherednik algebras and knot homologies. Here is the (very tentative) plan of the talks and some references. The group should start at the end of January.

(1) Introduction.

(2) Algebraic side: finite, affine, double affine Hecke algebras, degenerate and graded versions. [3], [8], [9].

(3) Classical Springer theory and graded affine Hecke algebra action. [2], [15].

(4) Affine Springer fibers, definitions and examples. [12], [14], [20].

(5) Global Springer action, action of the graded Cherednik algebra. [16], [21], [19], [22].

(6) Equivariant (co)homology of affine Springer fibers. [13], [5], [6], [7].

(7) Representations of Cherednik algebras and affine Springer theory. [18], [17].

(8) Hilbert schemes and Procesi bundles. [10], [11], [9].

(9) Hilbert schemes and Cherednik algebras. [4].

(10) Hilbert schemes and affine Springer fibers. [13], [1].

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